Example 2 on p 113

```
ClearAll["Global`*"]
```

```
hank = {y'''[x] - y''[x] + 100 y'[x] - 100 y[x] == 0,
   y[0] == 4, y'[0] == 11, y''[0] == -299}
dank = DSolve[hank, y[x], x]
```

$$\{-100\,y[x] + 100\,y'[x] - y''[x] + y^{(3)}[x] == 0,\ y[0] == 4,\ y'[0] == 11,\ y''[0] == -299\}$$

$$\{\{y[x] \to e^{x} + 3\,\text{Cos}[10\,x] + \text{Sin}[10\,x]\}\}$$

Above: This answer agrees with the text.

1 - 6 General solution
Solve the given ODE.

1. $y''' + 25\,y' = 0$

```
ClearAll["Global`*"]
```

```
jav = y'''[x] + 25 y'[x] == 0
nav = DSolve[jav, y[x], x]
```

$$25\,y'[x] + y^{(3)}[x] == 0$$

$$\left\{\left\{y[x] \to C[3] - \frac{1}{5}C[2]\,\text{Cos}[5\,x] + \frac{1}{5}C[1]\,\text{Sin}[5\,x]\right\}\right\}$$

1. Above: This answer agrees with the text.

3. $y^{iv} + 4\,y'' = 0$

```
ClearAll["Global`*"]
```

```
har = y''''[x] + 4 y''[x] == 0
mar = DSolve[har, y[x], x]
```

$$4\,y''[x] + y^{(4)}[x] == 0$$

$$\left\{\left\{y[x] \to C[3] + x\,C[4] - \frac{1}{4}C[1]\,\text{Cos}[2\,x] - \frac{1}{4}C[2]\,\text{Sin}[2\,x]\right\}\right\}$$

1. Above: This answer agrees with the text.

5. $\left(D^4 + 10\,D^2 + 9\,I\right)\,y = 0$

```
ClearAll["Global`*"]
```

```
yip = y''''[x] + 10 y''[x] + 9 y[x] == 0
nip = DSolve[yip, y[x], x]
```

$$9\,y[x] + 10\,y''[x] + y^{(4)}[x] == 0$$

```
{{y[x] → C[3] Cos[x] + C[1] Cos[3 x] + C[4] Sin[x] + C[2] Sin[3 x]}}
```

1. Above: This answer agrees with the text.

7 - 13 Initial value problem
Solve the IVP by a CAS, giving a general solution and the particular solution and its graph.

7. y''' + 3.2 y'' + 4.81 y' = 0, y[0] = 3.4, y'[0] = -4.6, y''[0] = 9.91

```
ClearAll["Global`*"]
```

First I can try to solve the general equation.

```
de = y''''[x] + 3.2 y''[x] + 4.81 y'[x] == 0
gs = DSolve[de, y[x], x]
```

$$4.81\,y'[x] + 3.2\,y''[x] + y^{(3)}[x] == 0$$

$$\{\{y[x] \to C[3] + e^{-1.6\,x}\,((-0.31185\,C[1] - 0.33264\,C[2])\,\text{Cos}[1.5\,x] + (-0.33264\,C[1] + 0.31185\,C[2])\,\text{Sin}[1.5\,x])\}\}$$

And make some substitutions for constants to help out the appearance a little.

```
gsf = gs /. {C[1] → 1, C[2] → 1, C[3] → 1}
```

$$\{\{y[x] \to 1 + e^{-1.6\,x}\,(-0.644491\,\text{Cos}[1.5\,x] - 0.02079\,\text{Sin}[1.5\,x])\}\}$$

Then I can try to solve the IVP.

```
de2 = {y''''[x] + 3.2 y''[x] + 4.81 y'[x] == 0,
   y[0] == 3.4, y'[0] == -4.6, y''[0] == 9.91}
```

$$\{4.81\,y'[x] + 3.2\,y''[x] + y^{(3)}[x] == 0,\ y[0] == 3.4,\ y'[0] == -4.6,\ y''[0] == 9.91\}$$

```
ps = DSolve[de2, y[x], x]
```

$$\{\{y[x] \to 2.4\,e^{-1.6\,x}\,(1.\,e^{1.6\,x} + 0.416667\,\text{Cos}[1.5\,x] - 0.833333\,\text{Sin}[1.5\,x])\}\}$$

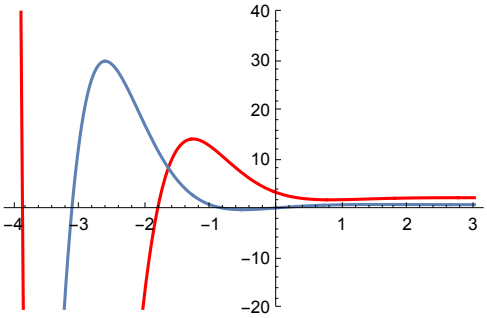
And ask for a slight modification to change the form.

```
trim = Expand[ps]
```

$$\{\{y[x] \to 2.4 + 1.\,e^{-1.6\,x}\,\text{Cos}[1.5\,x] - 2.\,e^{-1.6\,x}\,\text{Sin}[1.5\,x]\}\}$$

1. Above: The answer agrees with that of the text to 2S.

```
plot1 = Plot[y[x] /. ps, {x, -4, 3},
    PlotRange → {-20, 40}, PlotStyle → Red, ImageSize → 250];
plot2 = Plot[y[x] /. gsf, {x, -4, 3}, PlotRange → {-20, 40}];
Show[plot1, plot2]
```



2. Above: There was an odd gap at the max of gsf the first time it was plotted. Then the constant value of C[1] was jiggled and afterwards the gap disappeared.

9. 4 y''' + 8 y'' + 41 y' + 37 y = 0, y[0] = 9, y'[0] = -6.5, y''[0] = -39.75

In[1]:=
```
ClearAll["Global`*"]
```

First I can try to solve the general equation.

In[2]:=
```
gie = 4 y'''[x] + 8 y''[x] + 41 y'[x] + 37 y[x] == 0
gs = DSolve[gie, y[x], x]
```

Out[2]=
$$37\, y[x] + 41\, y'[x] + 8\, y''[x] + 4\, y^{(3)}[x] == 0$$

Out[3]=
$$\left\{\left\{y[x] \to e^{-x} C[3] + e^{-x/2} C[2] \text{Cos}[3 x] + e^{-x/2} C[1] \text{Sin}[3 x]\right\}\right\}$$

And make some substitutions for constants.

In[4]:=
```
gse = gs /. {C[1] → 1, C[2] → 1, C[3] → 1}
```

Out[4]=
$$\left\{\left\{y[x] \to e^{-x} + e^{-x/2} \text{Cos}[3 x] + e^{-x/2} \text{Sin}[3 x]\right\}\right\}$$

Then I can try to solve the IVP.

In[5]:=
```
pie = {4 y'''[x] + 8 y''[x] + 41 y'[x] + 37 y[x] == 0,
   y[0] == 9, y'[0] == -6.5, y''[0] == -39.75}
ps = DSolve[pie, y[x], x]
```

Out[5]=
$$\left\{37\, y[x] + 41\, y'[x] + 8\, y''[x] + 4\, y^{(3)}[x] == 0,\ y[0] == 9,\ y'[0] == -6.5,\ y''[0] == -39.75\right\}$$

Out[6]=
$$\left\{\left\{y[x] \to 5.\, e^{-x} \left(0.8 + 1.\, e^{x/2} \text{Cos}[3 x] + 6.09497 \times 10^{-18}\, e^{x/2} \text{Sin}[3 x]\right)\right\}\right\}$$

And alter it a little

In[7]:=
```
pse = Expand[ps]
```

Out[7]=
$$\left\{\left\{y[x] \to 4.\, e^{-x} + 5.\, e^{-x/2} \text{Cos}[3 x] + 3.04749 \times 10^{-17}\, e^{-x/2} \text{Sin}[3 x]\right\}\right\}$$

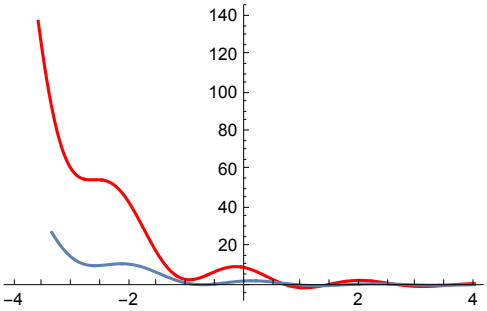
and a little more, until

In[8]:=
```
Chop[pse, 10^-16]
```

Out[8]=
$$\{\{y[x] \to 4.\, e^{-x} + 5.\, e^{-x/2} \text{Cos}[3\, x]\}\}$$

1. Above: The answer agrees with the text's.

```
plot1 = Plot[y[x] /. pse, {x, -4, 4},
    PlotRange → Automatic, PlotStyle → Red, ImageSize → 250];
plot2 = Plot[y[x] /. gse, {x, -4, 4}, PlotRange → Automatic];
Show[plot1, plot2]
```



11. $y^{iv} - 9\, y'' - 400\, y = 0,\ y[0] = 0,\ y'[0] = 0,\ y''[0] = 41,\ y'''[0] = 0$

```
ClearAll["Global`*"]
```

First I can try to solve the general equation.

```
nom = y''''[x] - 9 y''[x] - 400 y[x] == 0
gs = DSolve[nom, y[x], x]
```

$$-400\, y[x] - 9\, y''[x] + y^{(4)}[x] == 0$$

$$\{\{y[x] \to e^{-5\,x} C[3] + e^{5\,x} C[4] + C[1] \text{Cos}[4\, x] + C[2] \text{Sin}[4\, x]\}\}$$

And make some substitutions for constants.

```
gse = gs /. {C[1] → 1, C[2] → 1, C[3] → 1, C[4] → 1}
```

$$\{\{y[x] \to e^{-5\,x} + e^{5\,x} + \text{Cos}[4\, x] + \text{Sin}[4\, x]\}\}$$

Then I can try to solve the IVP.

```
nomp = {y''''[x] - 9 y''[x] - 400 y[x] == 0,
   y[0] == 0, y'[0] == 0, y''[0] == 41, y'''[0] == 0}
ps = DSolve[nomp, y[x], x]
```

$$\{-400\, y[x] - 9\, y''[x] + y^{(4)}[x] == 0,\ y[0] == 0,\ y'[0] == 0,\ y''[0] == 41,\ y^{(3)}[0] == 0\}$$

$$\{\{y[x] \to \frac{1}{2} e^{-5\,x} \left(1 + e^{10\,x} - 2\, e^{5\,x} \text{Cos}[4\, x]\right)\}\}$$

And alter it a little to improve the form

```
ps1 = ExpToTrig[ps]
```

$$\left\{\left\{y[x] \to \frac{1}{2}\,(\text{Cosh}[5\,x] - \text{Sinh}[5\,x])\,(1 - 2\,\text{Cos}[4\,x]\,\text{Cosh}[5\,x] + \text{Cosh}[10\,x] - 2\,\text{Cos}[4\,x]\,\text{Sinh}[5\,x] + \text{Sinh}[10\,x])\right\}\right\}$$

and alter it a little more

```
ps2 = Expand[ps1]
```

$$\left\{\left\{y[x] \to \frac{1}{2}\,\text{Cosh}[5\,x] - \text{Cos}[4\,x]\,\text{Cosh}[5\,x]^2 + \frac{1}{2}\,\text{Cosh}[5\,x]\,\text{Cosh}[10\,x] - \frac{1}{2}\,\text{Sinh}[5\,x] - \frac{1}{2}\,\text{Cosh}[10\,x]\,\text{Sinh}[5\,x] + \text{Cos}[4\,x]\,\text{Sinh}[5\,x]^2 + \frac{1}{2}\,\text{Cosh}[5\,x]\,\text{Sinh}[10\,x] - \frac{1}{2}\,\text{Sinh}[5\,x]\,\text{Sinh}[10\,x]\right\}\right\}$$

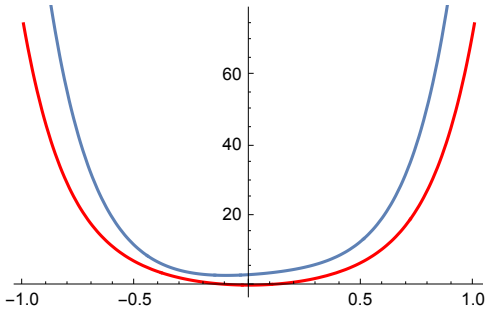
and maybe a little more, until

```
ps3 = Simplify[ps2]
```

```
{{y[x] → -Cos[4 x] + Cosh[5 x]}}
```

1. Above: The answer matches the text's.

```
plot1 = Plot[y[x] /. ps3, {x, -1, 1},
    PlotRange → Automatic, PlotStyle → Red, ImageSize → 250];
plot2 = Plot[y[x] /. gse, {x, -1, 1}, PlotRange → Automatic];
Show[plot1, plot2]
```



13. $y^{iv} + 0.45\,y''' - 0.165\,y'' + 0.0045\,y' - 0.00175\,y = 0$,
$y[0] = 17.4,\ y'[0] = -2.82,\ y''[0] = 2.0485,\ y'''[0] = -1.458675$

```
ClearAll["Global`*"]
```

First I can try to solve the general equation.

```
bi = y''''[x] + 0.45 y'''[x] - 0.165 y''[x] + 0.0045 y'[x] - 0.00175 y[x] == 0
gs = DSolve[bi, y[x], x]
```

$$-0.00175\,y[x] + 0.0045\,y'[x] - 0.165\,y''[x] + 0.45\,y^{(3)}[x] + y^{(4)}[x] == 0$$

$$\left\{\left\{y[x] \to e^{-0.7\,x}\,C[1] + e^{0.25\,x}\,C[4] + 1.\,C[3]\,\text{Cos}[0.1\,x] + 1.\,C[2]\,\text{Sin}[0.1\,x]\right\}\right\}$$

And make some substitutions for constants.

```
gse = gs /. {C[1] → 1, C[2] → 1, C[3] → 1, C[4] → 1}
```

$$\{\{y[x] \to e^{-0.7\,x} + e^{0.25\,x} + 1.\,\text{Cos}[0.1\,x] + 1.\,\text{Sin}[0.1\,x]\}\}$$

Then I can try to solve the IVP.

```
bip =
  {y''''[x] + 0.45 y'''[x] - 0.165 y''[x] + 0.0045 y'[x] - 0.00175 y[x] == 0,
   y[0] == 17.4, y'[0] == -2.82, y''[0] == 2.0485, y'''[0] == -1.458675}
ps = DSolve[bip, y[x], x]
```

$$\{-0.00175\,y[x] + 0.0045\,y'[x] - 0.165\,y''[x] + 0.45\,y^{(3)}[x] + y^{(4)}[x] == 0,\ y[0] == 17.4,\ y'[0] == -2.82,\ y''[0] == 2.0485,\ y^{(3)}[0] == -1.45868\}$$

$$\{\{y[x] \to 1.\,e^{-0.7\,x}\left(4.3 + 1.\,e^{0.95\,x} + 12.1\,e^{0.7\,x}\,\text{Cos}[0.1\,x] - 0.6\,e^{0.7\,x}\,\text{Sin}[0.1\,x]\right)\}\}$$

And alter it a little to improve the form

```
droop = Expand[ps]
```

$$\{\{y[x] \to 4.3\,e^{-0.7\,x} + 1.\,e^{0.25\,x} + 12.1\,\text{Cos}[0.1\,x] - 0.6\,\text{Sin}[0.1\,x]\}\}$$

1. Above: The answer matches the text's.

```
plot1 = Plot[y[x] /. droop, {x, -5, 5},
   PlotRange → {-100, 100}, PlotStyle → Red, ImageSize → 250];
plot2 = Plot[y[x] /. gse, {x, -5, 5}, PlotRange → Automatic];
Show[plot1, plot2]
```